ON THE WAVE DRAG OF NON ANIXYMMETRIC BODIES AT SUPERSONIC SPEEDS

(O VOLNOVOM SPOROTIVLENII NEOSESIMMETRICHNYKH TEL V SVERKHZVUKOVOM POTOKE)

PMM Vol.23, No.2, 1959, pp. 376-378

G.I. MAIKAPAR
(Moscow)

(Received 24 December 1958)

The bodies under consideration belong to a family of pyramids having polygonal cross sections with reentrant or concave corners. The surface of such a body is formed by stream surfaces behind plane shock waves, and the straight outer edges of the body appear as linear intersections of the plane shock waves. For such a class of bodies, a simple dependence of the wave drag on the geometrical characteristics is developed for given design conditions.

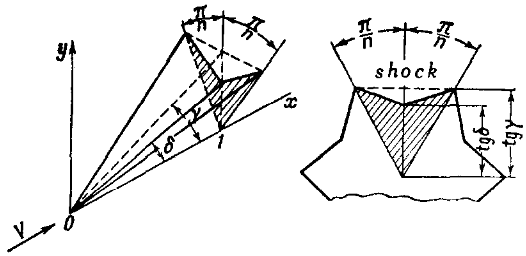

Fig. 1.

Heretofore the only class of bodies for which relatively simple and exact wave drag solutions have been obtained were circular cones. It is therefore of interest to consider other families of bodies for which the wave drag is easily found even though these families may be relatively narrow and the solutions for isolated "design" Mach numbers. Such a class of bodies is studied here. They are pyramids, with or without internal flow, having cross-sections in the form of reentrant polygons (see Figs. 1 and 4). The surface of such a body is formed by the stream surface behind a configuration of plane shock waves which intersect along (without extending beyond) the straight convex edges of the pyramids.*

Let us consider a body with a cross-section in the shape of a regular reentrant polygon (Fig. 1).

The velocity components behind the oblique shock wave are:

$$v_x = V\,[1-(1-\varepsilon)(\sin^2\gamma-\sin^2\omega)] \tag{1}$$

$$v_y = V\operatorname{ctg}\gamma\,(1-\varepsilon)(\sin^2\gamma-\sin^2\omega) \tag{2}$$

Here

$$\sin\omega=\frac{1}{M}=\frac{a_\infty}{V},$$

$$\varepsilon=\frac{c_p-c_v}{c_p+c_v}=\frac{1}{6}$$

From (1) and (2) we obtain the tangent of the angle between the body axis and the reentrant rib:

$$\frac{\operatorname{tg}\delta}{\operatorname{tg}\gamma}=\frac{\operatorname{ctg}^2\gamma\,(1-\varepsilon)(\sin^2\gamma-\sin^2\omega)}{1-(1-\varepsilon)(\sin^2\gamma-\sin^2\omega)}=\tau \tag{3}$$

The functional dependence $\tau=\tau(\gamma)$ is shown in Fig. 2. The area of the base cross section at $x=1$ is:

$$S=n\operatorname{tg}\frac{\pi}{n}\operatorname{tg}\gamma\operatorname{tg}\delta=n\operatorname{tg}\frac{\pi}{n}\,\frac{(1-\varepsilon)(\sin^2\gamma-\sin^2\omega)}{1-(1-\varepsilon)(\sin^2\gamma-\sin^2\omega)} \tag{4}$$

An "equivalent" circular cone with equal base area has a half nose angle θ_k such that

$$\operatorname{tg}\theta_k=\sqrt{\frac{n}{\pi}\operatorname{tg}\frac{\pi}{n}\,\frac{(1-\varepsilon)(\sin^2\gamma-\sin^2\omega)}{1-(1-\varepsilon)(\sin^2\gamma-\sin^2\omega)}} \tag{5}$$

The air pressure behind the shock is given by

$$p=(1-\varepsilon)\rho_\infty V^2\left(\sin^2\gamma-\frac{\varepsilon}{1+\varepsilon}\sin^2\omega\right)$$

and the pressure coefficient is

$$\bar{p}=\frac{p-p_\infty}{q_\infty}=2\,(1-\varepsilon)(\sin^2\gamma-\sin^2\omega) \tag{6}$$

From equations (5) and (6) follows a simple relation between the angle of the equivalent cone and the pressure coefficient (wave drag coefficient) which does not depend on Mach number:

---

* For constructing such bodies one may also use a group of plane shocks, for instance three shocks intersecting in a single point, and also suitable cylindrical shocks.

$$\operatorname{tg}\theta_k = \sqrt{\frac{n}{\pi}\operatorname{tg}\frac{\pi}{n}\frac{\bar{p}}{2-\bar{p}}} \tag{7}$$

or

$$\bar{p} = \frac{2\operatorname{tg}^2\theta_k}{\operatorname{tg}^2\theta_k + \frac{n}{\pi}\operatorname{tg}\frac{\pi}{n}} \tag{8}$$

Equation (8) facilitates comparison between the wave drag of the present class of bodies and that of the circular cones. One must bear in mind, however, that for each Mach number the body shape is different.

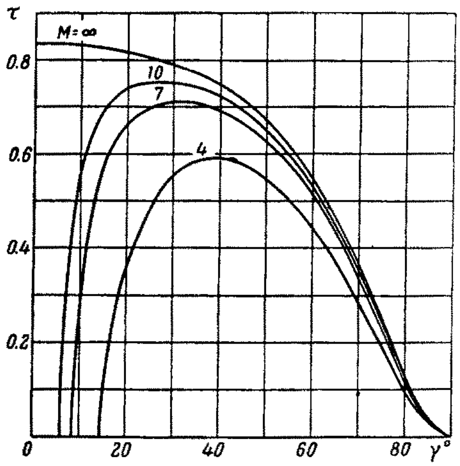

Fig. 2.

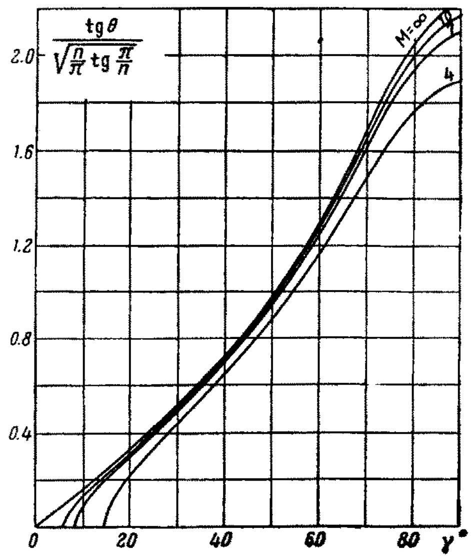

Fig. 3.

The drag coefficients, referred to that of a circular cone at $M = \infty$ as unity, are shown in Fig. 4. The upper set of curves labelled with the corresponding values of M refer to circular cones.

The same figure displays a series of cross sections of the pyramidal bodies equivalent in area to the circular cone with $\theta_k = 15^\circ$ at different Mach numbers.

The case of an asymmetric pyramidal body shown in Fig. 5. may be of interest. It resembles a (delta) wing with empennage. In this case the velocity components behind the shock wave are:

$$v_x = V\,[1-(1-\varepsilon)(\sin^2\gamma - \sin^2\omega)] \tag{9}$$

$$v_y = V\operatorname{ctg}\alpha\,(1-\varepsilon)(\sin^2\gamma - \sin^2\omega) \tag{10}$$

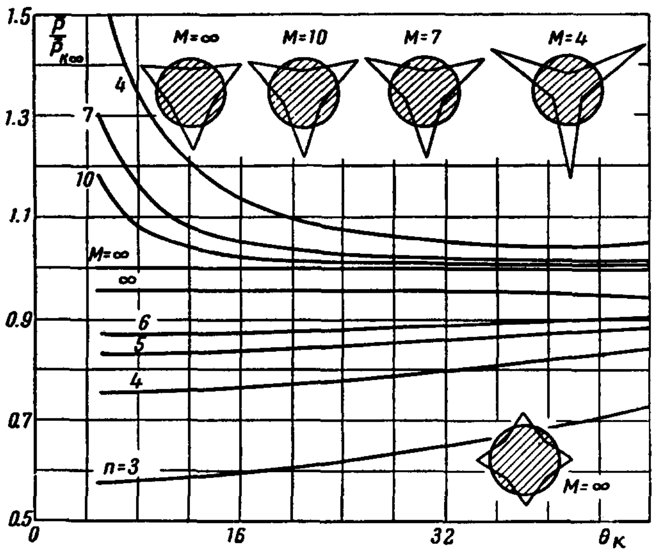

Fig. 4.

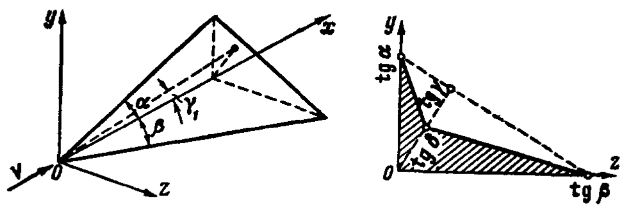

Fig. 5.

$$v_z = V \operatorname{ctg} \beta \, (1 - \varepsilon)(\sin^2 \gamma - \sin^2 \omega) \tag{11}$$

$$\sin \gamma = (1 + \operatorname{ctg}^2 \alpha + \operatorname{ctg}^2 \beta)^{-1/2}$$

The area of one quarter of the cross section at $x = 1$ (right side of Fig. 4) equals

$$S = \frac{1}{2}\left(\frac{\operatorname{tg} \alpha}{\operatorname{tg} \beta} + \frac{\operatorname{tg} \beta}{\operatorname{tg} \alpha}\right) \frac{(1 - \varepsilon)(\sin^2 \gamma - \sin^2 \omega)}{1 - (1 - \varepsilon)(\sin^2 \gamma - \sin^2 \omega)} \tag{12}$$

The equations (3) and (6) for tan δ and $\overline{p}$ remain valid. When $M \to \infty$ and $\epsilon \to 0$, the reentrant corners of the polygon straighten out.

Translated by M.V.M.